PHYC 463 Advanced Optics I
Fall 2007
Homework #1, Due Wednesday Aug. 29

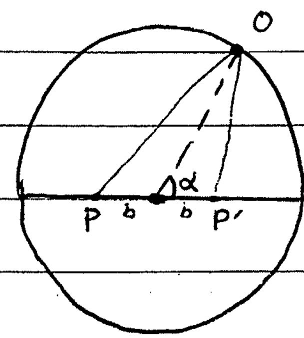
From Klein & Furtak

6' -Problem 1.13 (Fermat principle)
2' -Problem 1.15
6' -Problem 1.47
6' -Problem 1.49 (May approximate the room with a spherical volume)

- Pick up your graded homework (one-week after the due date) from its marked folder placed in the front office.
- The solution will be posted online and a copy will be also placed in a folder in the front office. Feel free to make photocopies if you need one.

HW # Solutions

1.13



(a) $\overline{OP}^2 = R^2 + b^2 + 2bR\cos\alpha$

$\overline{OP'}^2 = R^2 + b^2 - 2bR\cos\alpha$

optical path length (O.P.L) from P to P' is

$$(O.P.L) = \overline{OP} + \overline{OP'} = \sqrt{R^2+b^2+2bR\cos\alpha} + \sqrt{R^2+b^2-2bR\cos\alpha}$$

POP' is a true ray if (according to Fermat Principle)

$$\frac{d(O.P.L)}{d\alpha} = 0 \Rightarrow \frac{-bR\sin\alpha}{\sqrt{R^2+b^2+2bR\cos\alpha}} + \frac{bR\sin\alpha}{\sqrt{R^2+b^2-2bR\cos\alpha}} = 0$$

This is true if:

$\sin\alpha = 0 \Rightarrow \alpha = 0^\circ, 180^\circ \quad (b \neq 0)$

$\cos\alpha = 0 \Rightarrow \alpha = 90^\circ, 270^\circ \quad (b \neq 0)$

1.13 (Continue)

(b) $(O.P.L.)^2 = (PO + OP')^2$

$$= \left(\sqrt{R^2+b^2+2bR\cos\alpha} + \sqrt{R^2+b^2-2bR\cos\alpha}\right)^2$$

$$= R^2+b^2+2bR\cos\alpha + R^2+b^2-2bR\cos\alpha +$$

$$2\sqrt{(R^2+b^2)^2-(2bR\cos\alpha)^2}$$

$$= 2(R^2+b^2) + 2\left(R^4+b^4-2b^2R^2\cos 2\alpha\right)^{1/2}$$

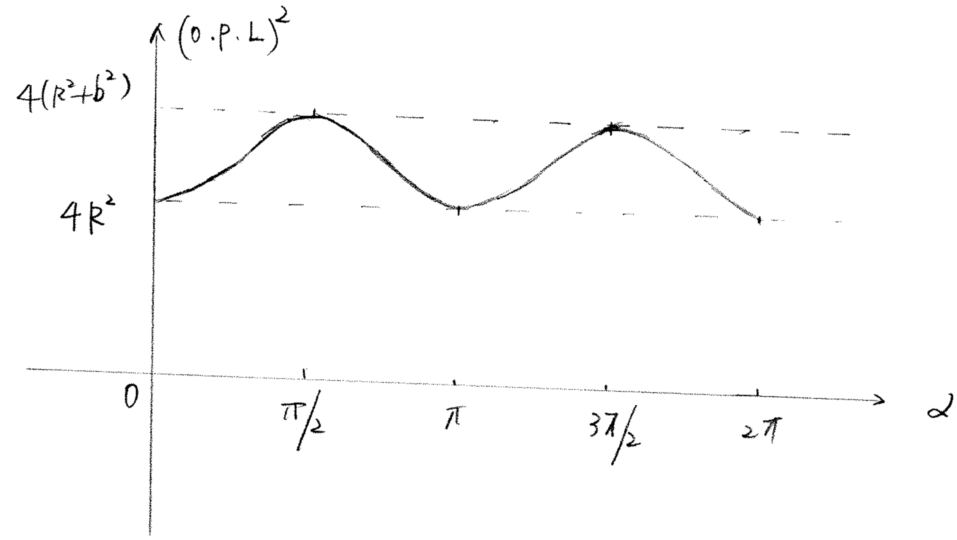
(c) when $\cos 2\alpha = 1$, $\alpha = 0, \pi$

$(O.P.L.)^2$ is at the minimum $4R^2$

when $\cos 2\alpha = -1$, $\alpha = \frac{\pi}{2}, \frac{3}{2}\pi$

$(O.P.L.)^2$ is at the maximum $4(R^2+b^2)$

$0 < \alpha < 2\pi$

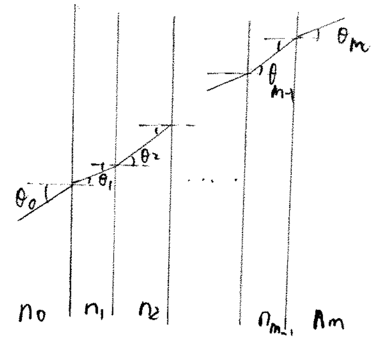


1.15

$$n_0 \sin\theta_0 = n_1 \sin\theta_1 = n_2 \sin\theta_2 = \cdots = n_m \sin\theta_m$$

$$\Rightarrow n_0 \sin\theta_0 = n_m \sin\theta_m$$

Thus the final refraction angle only depends on $\frac{n_m}{n_0}$ and θ_0



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$$\underbrace{\mathcal{E}}_{\text{Energy}} = \underbrace{p}_{\text{momentum}} c \Rightarrow p = \frac{\mathcal{E}}{c} \Rightarrow \langle p \rangle = \frac{\langle \mathcal{E} \rangle}{c}$$

$$I = \langle S \rangle = \langle u \rangle c = \frac{1}{2} c \epsilon_0 |E_0|^2 \Rightarrow$$ Energy per unit area per unit time

$$\Rightarrow \frac{\langle \text{momentum} \rangle}{\text{unit area} \cdot \text{unit time}} = \frac{\langle u \rangle c}{c} = \frac{I}{c}$$

* For totally absorbing sample

$$\frac{\langle \text{momentum} \rangle}{\text{unit time} \cdot \text{unit area}} = \frac{\text{Force}}{\text{unit area}} = \text{Pressure} = \frac{I}{c}$$

For a sample with surface reflectivity of R%, R% of photons undergo a momentum change of $2\hbar k$ (in the quantum picture). For the other photons, they maybe be absorbed or transmitted.

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1.47 Continue.

If the absorbed part is $\alpha\%$ of the rest of the photons, the total momentum change is

$$p = \frac{I}{c}\left(2 \times R\% + 1 \times (1-R\%) \times \alpha\% + 0 \times (1-R\%) \times (1-\alpha\%)\right)$$

when $\alpha = 0$, $R = 50$

$$p = \frac{I}{c} \times 2 \times R\% = \frac{1\,w/cm^2 \times 2}{3 \times 10^{10}\,cm/sec} \times 50\%$$

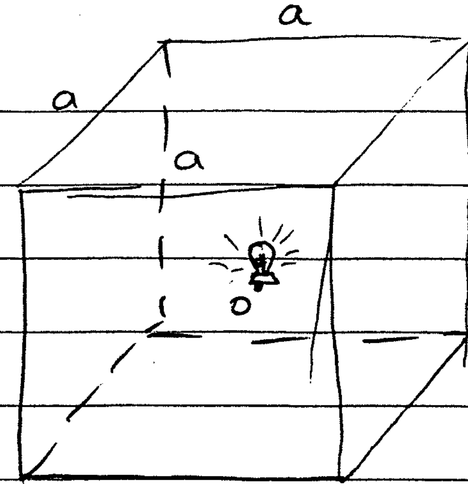
$$= 0.33 \times 10^{-10}\,J/cm^3$$

When $\alpha = 100$, $R = 50$

$$= \frac{I}{c} \times (1 + 0.5) = \frac{1\,w/cm^2 \times 1.5}{3 \times 10^{10}\,cm/sec}$$

$$= 0.5 \times 10^{-10}\,J/cm^3.$$

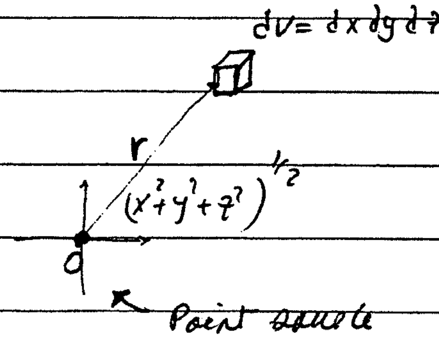
1.49



Average Energy density = $\langle u \rangle$

total energy in the room:

$$\mathcal{E}_{total} = \int_V \langle u \rangle \, dV$$



$$\mathcal{E}_{total} = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} \langle u(x,y,z) \rangle \, dx\,dy\,dz$$

$$\langle u(x,y,z) \rangle = \frac{P}{c\,4\pi r^2} = \frac{P}{4\pi c\,(x^2+y^2+z^2)}$$

$$\mathcal{E}_{total} = \frac{P}{4\pi c}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} \frac{dx\,dy\,dz}{x^2+y^2+z^2} \xrightarrow[\text{evaluation}]{\text{numerical}} \mathcal{E}_{total} \simeq \frac{P \times a \times 1.91}{4\pi c}$$

$$\boxed{\mathcal{E}_{total} = \frac{Pa}{c} \times 0.61}$$

Approximate solution: Assume the room is spherical with the same volume as the cube: $\frac{4\pi}{3}R^3 = a^3 \Rightarrow R = \left(\frac{4\pi}{3}\right)^{-1/3} a$

$dV = 4\pi r^2\,dr$

$$\langle u(r) \rangle = \frac{P}{c\,4\pi r^2} \qquad \mathcal{E}_{total} = \int_0^R \frac{P}{c\,\cancel{4\pi r^2}}\,\cancel{4\pi r^2}\,dr = \frac{PR}{c} = \frac{P}{c}\left(\frac{4\pi}{3}\right)^{-1/3} a$$

$$\mathcal{E}_{total} = \frac{Pa}{c} \times 0.62 = \frac{100 \times 3}{3 \times 10^8} \times 0.62 = 0.62\ \cancel{mJ}\ \mu J$$